



AP[®] Calculus BC

2014 Scoring Guidelines

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Question 1

Grass clippings are placed in a bin, where they decompose. For $0 \leq t \leq 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

- (a) Find the average rate of change of $A(t)$ over the interval $0 \leq t \leq 30$. Indicate units of measure.
- (b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.
- (c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \leq t \leq 30$.
- (d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

(a) $\frac{A(30) - A(0)}{30 - 0} = -0.197$ (or -0.196) lbs/day

1 : answer with units

(b) $A'(15) = -0.164$ (or -0.163)

The amount of grass clippings in the bin is decreasing at a rate of 0.164 (or 0.163) lbs/day at time $t = 15$ days.

2 : $\begin{cases} 1 : A'(15) \\ 1 : \text{interpretation} \end{cases}$

(c) $A(t) = \frac{1}{30} \int_0^{30} A(t) dt \Rightarrow t = 12.415$ (or 12.414)

2 : $\begin{cases} 1 : \frac{1}{30} \int_0^{30} A(t) dt \\ 1 : \text{answer} \end{cases}$

(d) $L(t) = A(30) + A'(30) \cdot (t - 30)$

$A'(30) = -0.055976$

$A(30) = 0.782928$

$L(t) = 0.5 \Rightarrow t = 35.054$

4 : $\begin{cases} 2 : \text{expression for } L(t) \\ 1 : L(t) = 0.5 \\ 1 : \text{answer} \end{cases}$

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Question 2

The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \leq \theta \leq \pi$.

(a) Let R be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of R .

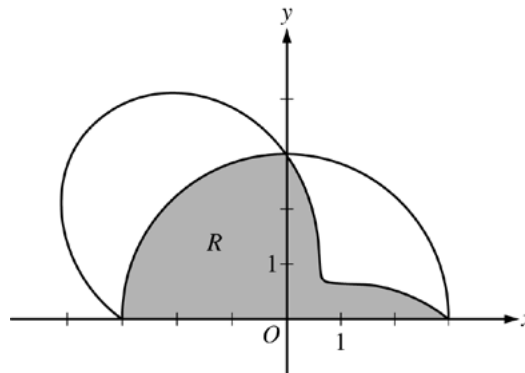
(b) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at

$$\theta = \frac{\pi}{6}.$$

(c) The distance between the two curves changes for $0 < \theta < \frac{\pi}{2}$.

Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.

(d) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \geq 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.



(a)
$$\text{Area} = \frac{9\pi}{4} + \frac{1}{2} \int_0^{\pi/2} (3 - 2\sin(2\theta))^2 d\theta$$

$$= 9.708 \text{ (or } 9.707)$$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(b)
$$x = (3 - 2\sin(2\theta))\cos\theta$$

$$\left. \frac{dx}{d\theta} \right|_{\theta=\pi/6} = -2.366$$

2 : $\begin{cases} 1 : \text{expression for } x \\ 1 : \text{answer} \end{cases}$

(c) The distance between the two curves is

$$D = 3 - (3 - 2\sin(2\theta)) = 2\sin(2\theta).$$

$$\left. \frac{dD}{d\theta} \right|_{\theta=\pi/3} = -2$$

2 : $\begin{cases} 1 : \text{expression for distance} \\ 1 : \text{answer} \end{cases}$

(d)
$$\frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt} = \frac{dr}{d\theta} \cdot 3$$

$$\left. \frac{dr}{dt} \right|_{\theta=\pi/6} = (-2)(3) = -6$$

2 : $\begin{cases} 1 : \text{chain rule with respect to } t \\ 1 : \text{answer} \end{cases}$

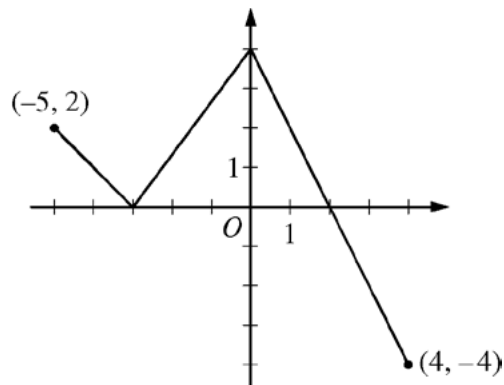
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Question 3

The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above.

Let g be the function defined by $g(x) = \int_{-3}^x f(t) dt$.

- (a) Find $g(3)$.
- (b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.
- (c) The function h is defined by $h(x) = \frac{g(x)}{5x}$. Find $h'(3)$.
- (d) The function p is defined by $p(x) = f(x^2 - x)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.



Graph of f

(a) $g(3) = \int_{-3}^3 f(t) dt = 6 + 4 - 1 = 9$

1 : answer

(b) $g'(x) = f(x)$

The graph of g is increasing and concave down on the intervals $-5 < x < -3$ and $0 < x < 2$ because $g' = f$ is positive and decreasing on these intervals.

2 : $\begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

(c) $h'(x) = \frac{5xg'(x) - g(x)5}{(5x)^2} = \frac{5xg'(x) - 5g(x)}{25x^2}$

3 : $\begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$

$$h'(3) = \frac{(5)(3)g'(3) - 5g(3)}{25 \cdot 3^2}$$

$$= \frac{15(-2) - 5(9)}{225} = \frac{-75}{225} = -\frac{1}{3}$$

(d) $p'(x) = f'(x^2 - x)(2x - 1)$

3 : $\begin{cases} 2 : p'(x) \\ 1 : \text{answer} \end{cases}$

$$p'(-1) = f'(2)(-3) = (-2)(-3) = 6$$

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Question 4

Train A runs back and forth on an east-west section of railroad track. Train A 's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	-120	-150

- (a) Find the average acceleration of train A over the interval $2 \leq t \leq 8$.
- (b) Do the data in the table support the conclusion that train A 's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.
- (c) At time $t = 2$, train A 's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A , in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.
- (d) A second train, train B , travels north from the Origin Station. At time t the velocity of train B is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time $t = 2$.

(a) average accel = $\frac{v_A(8) - v_A(2)}{8 - 2} = \frac{-120 - 100}{6} = -\frac{110}{3} \text{ m/min}^2$

1 : average acceleration

(b) v_A is differentiable $\Rightarrow v_A$ is continuous
 $v_A(8) = -120 < -100 < 40 = v_A(5)$

2 : $\begin{cases} 1 : v_A(8) < -100 < v_A(5) \\ 1 : \text{conclusion, using IVT} \end{cases}$

Therefore, by the Intermediate Value Theorem, there is a time t , $5 < t < 8$, such that $v_A(t) = -100$.

(c) $s_A(12) = s_A(2) + \int_2^{12} v_A(t) dt = 300 + \int_2^{12} v_A(t) dt$
 $\int_2^{12} v_A(t) dt \approx 3 \cdot \frac{100 + 40}{2} + 3 \cdot \frac{40 - 120}{2} + 4 \cdot \frac{-120 - 150}{2}$
 $= -450$

3 : $\begin{cases} 1 : \text{position expression} \\ 1 : \text{trapezoidal sum} \\ 1 : \text{position at time } t = 12 \end{cases}$

$s_A(12) \approx 300 - 450 = -150$

The position of Train A at time $t = 12$ minutes is approximately 150 meters west of Origin Station.

- (d) Let x be train A 's position, y train B 's position, and z the distance between train A and train B .

$z^2 = x^2 + y^2 \Rightarrow 2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$

$x = 300, y = 400 \Rightarrow z = 500$

$v_B(2) = -20 + 120 + 25 = 125$

$500 \frac{dz}{dt} = (300)(100) + (400)(125)$

$\frac{dz}{dt} = \frac{80000}{500} = 160 \text{ meters per minute}$

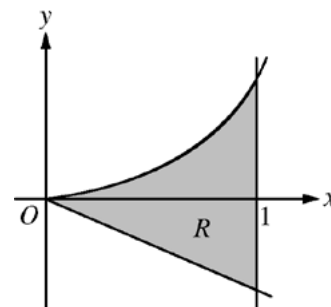
3 : $\begin{cases} 2 : \text{implicit differentiation of} \\ \text{distance relationship} \\ 1 : \text{answer} \end{cases}$

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Question 5

Let R be the shaded region bounded by the graph of $y = xe^{x^2}$, the line $y = -2x$, and the vertical line $x = 1$, as shown in the figure above.

- (a) Find the area of R .
- (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = -2$.
- (c) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of R .



$$\begin{aligned} \text{(a) Area} &= \int_0^1 (xe^{x^2} - (-2x)) dx \\ &= \left[\frac{1}{2}e^{x^2} + x^2 \right]_{x=0}^{x=1} \\ &= \left(\frac{1}{2}e + 1 \right) - \frac{1}{2} = \frac{e+1}{2} \end{aligned}$$

$$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$$

$$\text{(b) Volume} = \pi \int_0^1 \left[(xe^{x^2} + 2)^2 - (-2x + 2)^2 \right] dx$$

$$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$$

$$\text{(c) } y' = \frac{d}{dx}(xe^{x^2}) = e^{x^2} + 2x^2e^{x^2} = e^{x^2}(1 + 2x^2)$$

$$\text{Perimeter} = \sqrt{5} + 2 + e + \int_0^1 \sqrt{1 + [e^{x^2}(1 + 2x^2)]^2} dx$$

$$3 : \begin{cases} 1 : y' = e^{x^2}(1 + 2x^2) \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

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Question 6

The Taylor series for a function f about $x = 1$ is given by $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^n}{n} (x-1)^n$ and converges to $f(x)$ for $|x-1| < R$, where R is the radius of convergence of the Taylor series.

- (a) Find the value of R .
- (b) Find the first three nonzero terms and the general term of the Taylor series for f' , the derivative of f , about $x = 1$.
- (c) The Taylor series for f' about $x = 1$, found in part (b), is a geometric series. Find the function f' to which the series converges for $|x-1| < R$. Use this function to determine f for $|x-1| < R$.

- (a) Let a_n be the n th term of the Taylor series.

$$\begin{aligned} \frac{a_{n+1}}{a_n} &= \frac{(-1)^{n+2} 2^{n+1} (x-1)^{n+1}}{n+1} \cdot \frac{n}{(-1)^{n+1} 2^n (x-1)^n} \\ &= \frac{-2n(x-1)}{n+1} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left| \frac{-2n(x-1)}{n+1} \right| = 2|x-1|$$

$$2|x-1| < 1 \Rightarrow |x-1| < \frac{1}{2}$$

The radius of convergence is $R = \frac{1}{2}$.

- (b) The first three nonzero terms are

$$2 - 4(x-1) + 8(x-1)^2.$$

The general term is $(-1)^{n+1} 2^n (x-1)^{n-1}$ for $n \geq 1$.

- (c) The common ratio is $-2(x-1)$.

$$f'(x) = \frac{2}{1 - (-2(x-1))} = \frac{2}{2x-1} \text{ for } |x-1| < \frac{1}{2}$$

$$f(x) = \int \frac{2}{2x-1} dx = \ln|2x-1| + C$$

$$f(1) = 0$$

$$\ln|1| + C = 0 \Rightarrow C = 0$$

$$f(x) = \ln|2x-1| \text{ for } |x-1| < \frac{1}{2}$$

3 : $\begin{cases} 1 : \text{sets up ratio} \\ 1 : \text{computes limit of ratio} \\ 1 : \text{determines radius of convergence} \end{cases}$

3 : $\begin{cases} 2 : \text{first three nonzero terms} \\ 1 : \text{general term} \end{cases}$

3 : $\begin{cases} 1 : f'(x) \\ 1 : \text{antiderivative} \\ 1 : f(x) \end{cases}$